Supplemental Appendix to:

A Political Economy of International Organizations

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1 Appendix

1.1 Notation

Table A1: Notation

Variable	Interpretation	Detail
Key State Variables		
α	Vote share in IO for hegemon	$\alpha \in (0,1)$
κ	Share of cost paid by members	$\kappa \in (0,1)$
θ	Development value of the project	$\theta \sim N\left(\mu, \frac{1}{\delta}\right)$
ω	Political value of the project to H	$Pr(\omega \leq z) = W(z)$
Strategies		
r	A's recommendation	$r \in \{0,1\}$
v_i	Vote to fund by member i	$v_i \in \{0,1\}$
v_H	Vote to fund by H	$v_H \in \{0,1\}$
Signals and Prior		
s_i	member i's signal of development value	$s_i \sim \left(\theta, \frac{1}{\delta_m}\right)$
s_A	A's signal of development value	$s_A \sim \left(\theta, \frac{1}{\delta_A}\right)$
μ	Prior on development value	$\theta \sim N\left(\mu, \frac{1}{\delta}\right)$
Payoffs		
ψ	Bureaucratic value of project	$\psi > 0$
ρ	Reputational cost to A	$\rho > 0$
c	Operating cost to A	$c > 0$
Parameters		
M	Number of members	$M > 1$
γ	Financial capacity of H relative to M	$\gamma > 0$
μ, δ	Prior mean and precision on θ	$\mu \in \mathbb{R}, \delta \in \mathbb{R}_+$
δ_m	Precision of member i's signal	$\delta \in \mathbb{R}_+$
δ_A	Precision of A's signal	$\delta_A \in \mathbb{R}_+$

To economize notation, we introduce the following:

- $\bar{\theta} = \frac{\kappa\gamma}{M}$
- $\Delta = (\delta + \delta_m + \delta_A)$
- $\widehat{s_i(s_A)} = \frac{1}{\delta_m}\left[\Delta\bar{\theta} - \delta\mu - \delta_A s_A\right]$

1.2 Proofs

Proof of Proposition ??: The members' and hegemon's best-response voting strategies were derived in the main text, and restated here:

$$v_H = \mathbb{1}[\omega \geq 1 - \kappa]$$
$$v_i = \mathbb{1}[s_i \geq \widehat{s_i(s_A)}], \quad \text{where } \widehat{s_i(s_A)} = \frac{1}{\delta_m}\left[(\delta + \delta_m + \delta_A)\frac{\kappa\gamma}{M} - \delta\mu - \delta_A s_A\right]$$

For notational convenience, let $y \in \{0, 1\}$ denote whether a project is funded. Aggregating the members' and the hegemon's votes, we have that

$$y = \mathbb{1}\left[v_H\alpha + \frac{(1-\alpha)}{M}\sum_{i=1}^{M} v_i \geq \frac{1}{2}\right]$$

as per Equation (??). Also for notational convenience, let $\widehat{s_i} = \widehat{s_i(s_A)}$ Applying Assumption ??, and considering a large M, we can apply the Weak Law of Large Numbers to show that empirical distribution of the members' signals converges to the population distribution, and thus that the fraction of members that vote yes converges to $Pr(s_i > \widehat{s_i}|\theta)$, which is equal to $\Phi\left(\sqrt{\delta_M}(\theta - \widehat{s_i}\right)$ Thus we can rewrite the vote aggregation and project approval decision as follows:

$$y = 1 \iff \alpha v_H + (1-\alpha)Pr(s_i > \widehat{s_i}|\theta) > \frac{1}{2}$$

Given $Pr(s_i > \widehat{s_i}|\theta) = \Phi\left(\sqrt{\delta_m}(\theta - \widehat{s_i})\right)$, and substituting for $\widehat{s_i}$ and rearranging, we have that $y = 1$ if and only if

$$\theta > \frac{1}{\delta_m}\left[\Delta\bar{\theta} - \delta\mu - \delta_A s_A\right] + \frac{1}{\sqrt{\delta_m}}\Phi^{-1}\left(\frac{1 - 2\alpha v_H}{2 - 2\alpha}\right) \equiv \theta_{v_H} \tag{1}$$

Given this voting behavior, we now consider the decision of the IO to recommend the project or not.

To begin, recall that A's recommendation decision is made before H's vote is cast, but after H has declared its vote intention. Let $\widehat{v_H} \in \{0, 1\}$ denote a conjecture by A as to whether or not H will vote yes. A's conjecture implies that, given θ, a recommended project will be approved iff

$$\theta > \theta_{\widehat{v_H}} = \frac{1}{\delta_m}\left[\Delta\bar{\theta} - \delta\mu - \delta_A s_A\right] + \frac{1}{\sqrt{\delta_m}}\Phi^{-1}\left(\frac{1 - 2\alpha\widehat{v_H}}{2 - 2\alpha}\right)$$

Of course A also does not know θ when she makes her recommendation decision. Rather, she has a posterior belief of θ given her private signal and the common prior, which is distributed

$$\theta|s_A \sim N\left(\frac{\delta\mu + \delta_A s_A}{\delta + \delta_A}, \frac{1}{\delta + \delta_A}\right)$$

Thus given conjecture $\widehat{v_H}$, she believes that the probability that the project will be funded, if recommended, is

$$Pr(y=1|r=1,s_A,\widehat{v_H}) = Pr(\theta > \theta_{\widehat{v_H}}|s_A) = \Phi\left(\sqrt{\delta+\delta_A}\left(\frac{\delta\mu+\delta_A s_A}{\delta+\delta_A} - \theta_{\widehat{v_H}}\right)\right)$$

Restating Equation (**??**) in terms of A's conjecture $\widehat{v_H}$, we can express A's decision to recommend a project as:

$$r = 1 \iff Pr(y=1|r=1,s_A,\widehat{v_H}) > \frac{c+\rho}{\psi+\rho}$$

Substituting, we have $\sqrt{\delta+\delta_A}\left(\frac{\delta\mu+\delta_A s_A}{\delta+\delta_A} - \theta_{\widehat{v_H}}\right) > \Phi^{-1}\left(\frac{c+\rho}{\psi+\rho}\right)$, which rearranges to

$$s_A > -\frac{\delta\mu}{\delta_A} + \frac{\delta+\delta_A}{\delta_A}\bar{\theta} + \frac{\delta_m(\delta+\delta_A)}{\Delta\delta_A}\left[\frac{1}{\sqrt{\delta+\delta_A}}\Phi^{-1}\left(\frac{c+\rho}{\psi+\rho}\right) + \frac{1}{\sqrt{\delta_m}}\Phi^{-1}\left(\frac{1-2\alpha\widehat{v_H}}{2-2\alpha}\right)\right] \equiv s^*_{\widehat{v_H}}$$

which provides the threshold values in (**??**) and (**??**).

So altogether, given conjecture $\widehat{v_H}$, A's recommendation strategy is given by

$$r = 1 \iff s_A > s^*_{\widehat{v_H}} \tag{2}$$

Further, we can see that

$$s^*_1 < s^*_0 \tag{3}$$

meaning that $Pr(r=1|\widehat{v_H}=1) > Pr(r=1|\widehat{v_H}=0)$.

Now we turn to H's declaration strategy. Let $\chi(d)$ denote the probability that A assigns to H playing $v_H = 1$ given H's announcement $d \in \{0,1\}$. Given belief χ, A will play a threshold strategy of $r = 1 \iff s_A > s^*_\chi$, where s^*_χ is a convex combination of s^*_0 and s^*_1 when $\chi \in (0,1)$. If $s^*_{\chi(d')} = s^*_{\chi(d'')}$ for $d' \neq d''$, then A is ignoring H's message, and H can do no better than to randomize his messages independently of ω (i.e. babbling). If on the other hand $s^*_{\chi(d')} > s^*_{\chi(d'')}$, then we have that $Pr(r=1|d'') > Pr(r=1|d')$. Since H unambiguously prefers to encourage A's recommendations when $\omega > 1-\kappa$ and to discourage otherwise, it follows that H will send message d'' if $\omega > 1-\kappa$, and send message d' otherwise. This is of course the same rule governing H's voting decision given a recommendation. The meaning of the messages is arbitrary, so we can assign $d = 0$ to the message that decreases the probability of recommendation, and $d = 1$ to the message that increases it. In equilibrium, H's vote matches his announcement and A's conjecture is always correct: $\chi(d) = \widehat{v_H} = v_H = d$ for $d = 0, 1$. ■

Proof of Corollary ??: For the first inequality: by A's recommendation strategy, $E[\theta|r=1] = E[\theta|s_A > s^*_{v_H}]$ and $E[\theta|r=0] = E[\theta|s_A < s^*_{v_H}]$. Given that $E[\theta|s_A]$ is increasing in s_A it follows immediately from standard properties of truncated distributions that $E[\theta|r=1] > E[\theta|r=0]$.

For the second inequality: Denote $\hat{\omega} = 1-\kappa$, so that $v_H = \mathbb{1}[\omega > \hat{\omega}]$. From A's recommendation strategy and H's declaration strategy as given in Proposition **??**, we have:

$$r = \begin{cases} 1, & s_A > s^*_0 \\ 1, & s_A \in (s^*_1, s^*_0) \text{ and } \omega > \hat{\omega} \\ 0 & otw \end{cases}$$

By the law of total expectation we have that

$$E[\omega|r=1]=(1-\pi_1)E[\omega|s_A>s_0^*]+\pi_1E[\omega|s_A\in(s_1^*,s_0^*),\omega>\hat{\omega}]=(1-\pi_1)E[\omega]+\pi_1E[\omega|\omega>\hat{\omega}]$$

and

$$E[\omega|r=0]=(1-\pi_2)E[\omega|s_A<s_0^*]+\pi_2E[\omega|s_A\in(s_1^*,s_0^*),\omega<\hat{\omega}]=(1-\pi_2)E[\omega]+\pi_2E[\omega|\omega<\hat{\omega}]$$

for some $\pi_1,\pi_2\in(0,1)$. It follows that

$$E[\omega|r=1]-E[\omega|r=0]=\pi_1(E[\omega|\omega>\hat{\omega}]-E[\omega])+\pi_2(E[\omega]-E[\omega|\omega<\hat{\omega}])$$

From standard properties of truncated distributions, we know that this quantity is strictly positive. ■

Proof of Corollary ?? : By A's recommendation strategy, and by independence of s_A and ω, we have $E[\theta|r=1,v_H=1]=E[\theta|s_A>s_1^*]$ and $E[\theta|r=1,v_H=0]=E[\theta|s_A>s_0^*]$. Given that $E[\theta|s_A]$ is increasing in s_A, and given that $s_1^*<s_0^*$, it follows from standard properties of truncated distributions that $E[\theta|s_A>s_1^*]<E[\theta|s_A>s_0^*]$. ■

Proof of Corollary ??: By Equation (1), and by independence of ω and θ, we have that $E[\theta|funded,v_H]=E[\theta|\theta>\theta_{v_H}]$, and that $\theta_1<\theta_0$. Again by standard properties of truncated distributions it follows immediately that $E[\theta|\theta>\theta_1]<E[\theta|\theta>\theta_0]$. ■

Proof of Proposition ??: $\frac{d\widehat{s_i(s_A)}}{d\kappa}>0$ follows directly from differentiation of (??). $\frac{ds_0^*}{d\kappa}=\frac{ds_1^*}{d\kappa}>0$ follows directly from differentiation of (??) and (??), which in turn implies $\frac{dPr[r=1|v_H=1]}{d\kappa}<0$ and $\frac{d\Pr[r=1|v_H=0]}{d\kappa}<0$, because $Pr[r=1|v_H]=Pr(s_A>s_{v_H}^*)$. ■

Proof of Proposition ??: Differentiating equations (??) and (??) with respect to α gives

$$\frac{ds_{v_H}^*}{d\alpha}=\frac{\sqrt{\delta_m}(\delta+\delta_A)}{\Delta\delta_A}\frac{1}{\phi\left(\Phi^{-1}\left(\frac{1-2\alpha v_H}{2-2\alpha}\right)\right)}\frac{(1-2v_H)}{(1-\alpha)^2(2)} \tag{4}$$

which shows that $\frac{ds_0^*}{d\alpha}>0$ and $\frac{ds_1^*}{d\alpha}<0$. The derivatives $\frac{d\Pr[r=1|v_H=1]}{d\alpha}>0$, $\frac{d\Pr[r=1|v_H=0]}{d\alpha}<0$ follow immediately from the fact that $Pr(r=1|v_H)=Pr(s_A>s_{v_H}^*)$. ■

Proof of Proposition ??: The first claim, regarding the signs of the second derivatives, follows directly from differentiation of Equation (4). The second claim, that as $\delta_A\to\infty$, $s_0^*\to\frac{\kappa\gamma}{M}\leftarrow s_1^*$, follows directly from (??) and (??). ■

Proof of Proposition ??: Consider each point of the proposition in turn.

1. From (??) and (??), we see that as $\alpha\to\frac{1}{2}$, we have $s_0^*\to\infty$ and $s_1^*\to-\infty$. This means that A recommends a project if and only if H supports it. Likewise, from (??), we see that with $\alpha\to\frac{1}{2}$, a recommended project is approved if and only if H supports it. Thus λ portion of projects are recommended and funded, each bringing H an expected benefit of η and a cost of $1-\kappa$. Because project recommendation and approval is independent of developmental value, a member's expected benefit of a project is simply μ, the prior expectation of developmental value, and each comes at a cost $\frac{\kappa\gamma}{M}$.

2. Proposition **??** showed that as $\delta_A \to \infty$, $s_0^* \to \frac{\kappa\gamma}{M} \leftarrow s_1^*$. This means that regardless of H's support or opposition, A recommends projects when it receives a signal $s_A \geq \frac{\kappa\gamma}{M}$. Given this recommendation threshold, from (**??**), we see that as $\delta_A \to \infty$, we have $\widehat{s_i(s_A)} \to -\infty$, meaning that all members vote in favor of any project A recommends. Thus the portion of recommended and funded projects is simply the portion with developmental value greater than $\frac{\kappa\gamma}{M}$, that is, $\Phi\left(\sqrt{\delta}\left(\mu - \frac{\kappa\gamma}{M}\right)\right)$. By standard properties of the truncated normal distribution, the expected developmental value of these funded projects is $\mu + \frac{1}{\sqrt{\delta}} \frac{\phi\left(\sqrt{\delta}\left(\mu - \frac{\kappa\gamma}{M}\right)\right)}{\Phi\left(\sqrt{\delta}\left(\mu - \frac{\kappa\gamma}{M}\right)\right)}$. Whether or not a project gets funded is independent of H's preference, $E[\omega|funded] = E[\omega]$, so H's expected political value of funded projects is $\lambda\eta - \eta(1-\lambda)$, with each carrying a cost of $1-\kappa$.

■